Conversion of Chord Length Data into Bubble Size Distribution: Generation of Chord Length Data and the Methodology Comparison

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1. Introduction

Bubble size and its distribution play an important role in thermal hydrodynamic processes in multiphase flow systems. By using the conductivity or optical probe techniques, the size and distribution of bubbles can only be inferred indirectly from a measured chord length data (CLD). Some methods are proposed to convert a CLD into the bubble size distribution (BSD), and they can be classified into parametric, semi-parametric and non-parametric. Most of methods are derived from the following relation of the conditional probability functions that are established under the geometric constraints:

$$P(y)=\int_0^\infty P(R)P(y\mid R)\,dR \qquad (1)$$

where $P(R)$ is PDF of bubbles of all sizes R pierced by a probe, and $P(y|R)$ is PDF of chord length y corresponding to bubbles of a specified size R. These methods are limited to flows of bubbles having symmetric shapes, i.e. spherical, ellipsoidal, or cap-spherical. Although the methods were developed from a common relation, there are no physical bases as well as the lack of experimental data to validate them.

In this work, the CLD is generated for comparing different conversion methods. The range of bubble size is determined by the Hinze's theory. The CLDs are applied to numerical backward transforms (NBT), analytical backward transform (ABT), and analytical semi-parametric method using Parzen window estimator (ParzenES) to obtain the BSD. A comparison for the obtained results is performed.

2. The generation of chord length data

It is stated in literatures that the BSDs obey some kinds of probability functions, i.e. normal, lognormal, or gamma functions. The distributions can be single or multiple peaks, depending on the bubble interaction processes. The position of the peaks changes slightly with flow conditions. It suggests using the probability functions to generate CLDs whose distribution contains one or two peaks located at positions being proportional linearly to maximum bubble size.

The shape and size of bubbles depend on the forces acting on the bubbles, viscous stresses and dynamic pressure, which are expressed in the form of dimensionless groups such as Weber, Viscous and Reynolds numbers. The maximum bubble size d_m obtains at a critical Weber number at which bubbles will be broken up. Hinze established an expression of d_m for isotropic turbulent flows as follows:

$$d_m = 0.725\left(\rho_L/\sigma\right)^{-0.6}\varepsilon^{-0.4} \qquad (2)$$

where ε is energy dissipation rate per unit mass, ρ_L is liquid density and σ is surface tension. The generation procedure consists of (1) calculation of d_m, (2) generation of two PDFs and summation, (3) calculation of CDF, and (4) generation of a set of random numbers assigned to chord length value to find chord length probabilities.

3. The methods of conversion of chord length distribution into bubble size distribution

The shapes of bubbles assumed are spherical and ellipsoid for ABT and ParzenES methods, whereas a truncated-ellipsoid shape is assumed for NBT method.

3.1 Numerical backward transform

The range of chord length value is divided into some equal segments, and the probability in each segment is expressed as follows (Clark 1988):

$$P(y_i<y<y_{i+1})\simeq\sum_{j=0}^{m-1}\left[\int_{y_i}^{y_{i+1}}P(y\mid R)\,dy\right]P(R_j)\Delta R \qquad (3)$$

where

$$P(y\mid R)=\begin{cases} y/2\alpha^2R^2 & 0\le y<2\alpha QR \\ 2(y-\alpha RQ)/\alpha^2R^2 & 2\alpha QR\le y<\alpha R(1+Q) \\ 0 & \text{otherwise}\end{cases}$$

The $P(R)$ can be easily obtained by eq. (3) for a known chord length data.

3.2 Analytical backward transform

Differentiating both sides of eq. (1) under the assumption of ellipsoid-shaped bubbles gives

$$P_b(R)=\alpha\left[P_c(2\alpha R)-2\alpha RP_c^{'}(2\alpha R)\right] \qquad (4)$$

where $P(y\mid R)=y/2\alpha^2R^2,\quad R=y/2\alpha$.

The CLD is fitted with a probability function to find $P_c(y)$, and then substitute into equation 4 to find $P_b(R)$.

3.3 Analytical and Semi-parametric method using Parzen Window Estimator

The $P_c(y)$ can be evaluated from a set of chord length values using the following Parzen window estimator,

$$P_c(y) = \frac{1}{nh\sqrt{2\pi}} \sum_{j=1}^{n} e^{-(y-Y_j)^2/2h^2} \qquad (5)$$

where the Parzen window width is determined by maximizing the measure of performance $J_{\log}$ given by

$$J_{log} = \sum_{j=1}^{n} \log\left(P_b\left(2\alpha Y_j\right)\right) \qquad (6)$$

or by Monte-Carlo simulations

$$h = n^{-1/5}\sqrt{y_{mean} \times y_{std}} \qquad (7)$$

and $P_b(y)$ is determined by substitute eq. (5) into eq. (4).

3. Results and Discussion

The sensitivity evaluation of maximum bubble diameter on the flow condition indicates that the main influence comes from liquid velocity J_f. The bubble diameter decreases very quickly with increasing J_f (fig. 1). The relationship between the dimensionless groups shows the competition of deformation forces and restoring forces. The critical Weber number establishes the balance between viscous stresses and surface tension force. Beyond the value, bubbles will be broken up. The value seems to be constant since the flow velocity increases while diameter decreases. The effect of dynamic pressure on the deformation of bubbles is insignificant when compared with the others.

The range of equivalent bubble diameter is larger than that of chord length because of the assumption of bubble shape. Fig. 2 shows a good agreement among the methods, especially between ABT and ParzenES methods. The shape of BSDs obtained by these methods for a CLD is identical and similar to the shape of chord distributions. However, there is a significant difference in the region of larger bubble sizes, which induces the difference of the high of peaks. In the case of NBT, the rough tail of BSDs indicates the significant error coming from the propagation of errors from the region of small bubbles in the deduction process. Additional, the NBT is limited to a certain number of intervals of chord length value; more intervals will cause lager error.

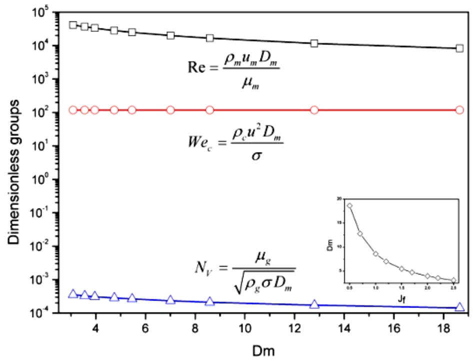

Fig. 1 The effect of dimensionless groups

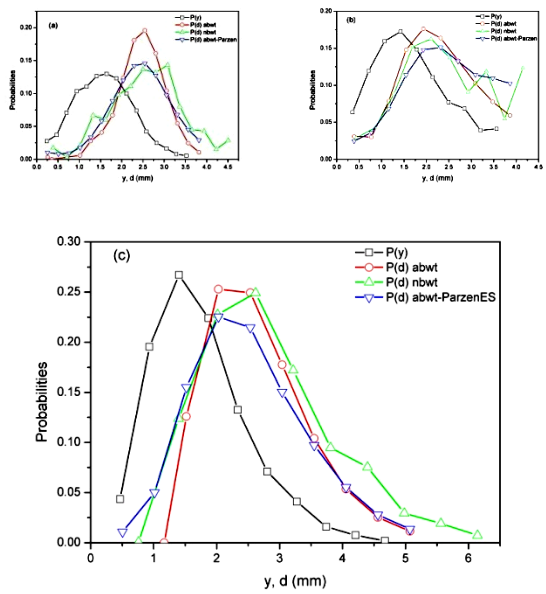

Fig. 2 Bubble size distribution with (a) normal, (b) lognormal, and (c) gamma CLDs

The CLDs in the form of normal, lognormal or gamma distributions of single or two peaks can apply to the methods quite well. The normal distribution is mostly suitable for describing the CLD of small bubbles. The set of chord values is used instead of chord distribution for the ParzenES. The number of intervals should be small enough to eliminate the errors because probabilities depend on the statistic number of bubbles. The probability becomes zero if interval is too small. Thus, the NBT can apply only for a certain interval.

4. Conclusion

The CLD is generated in the different forms by a simple procedure using the Hinze's theory. The conversion methods show a good agreement. However, some limitations remain for each method. In some cases, the difference can be significant because of assumptions of bubble shape and the error propagation.

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